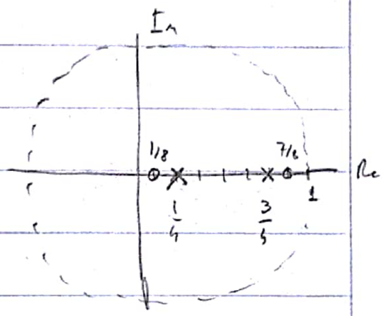
$$x[n] = \underbrace{5\left(1/8\right)^n u[n]}\ -\ \underbrace{\left(\tfrac{7}{8}\right)^n u[-n-1]}$$

$$X(z) = 5\cdot\frac{1}{1-1/8\,z^{-1}} + \frac{1}{1-7/8\,z^{-1}} = \frac{5-\frac{35}{8}z^{-1}+1-1/8\,z^{-1}}{(1-1/8\,z^{-1})(1-7/8\,z^{-1})} = \frac{6-\frac{36}{8}z^{-1}}{(1-1/8\,z^{-1})(1-7/8\,z^{-1})} = \frac{6\left(1-\frac{3}{4}z^{-1}\right)}{(\ )(\ )}$$

$|z|>1/8$ $\qquad$ $|z|<7/8$

$$\Rightarrow\quad \boxed{X(z) = \frac{6\left(1-3/4\,z^{-1}\right)}{(1-1/8\,z^{-1})(1-7/8\,z^{-1})}\,,\quad \frac{1}{8}<|z|<\frac{7}{8}}$$



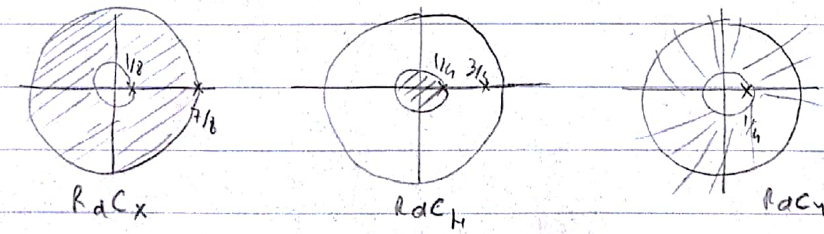
$$y[n] = 6\left(\tfrac{1}{4}\right)^n u[n] \ \longrightarrow\ \boxed{Y(z) = \frac{6}{1-1/4\,z^{-1}}\qquad |z|>1/4}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\cancel{6}}{(1-1/4\,z^{-1})}\cdot\frac{(1-1/8\,z^{-1})(1-7/8\,z^{-1})}{\cancel{6}(1-3/4\,z^{-1})} \Rightarrow \boxed{H(z) = \frac{(1-1/8\,z^{-1})(1-7/8\,z^{-1})}{(1-1/4\,z^{-1})(1-3/4\,z^{-1})}}\quad \text{RdC ??}$$

<u>Propiedad</u>: $\left.\begin{array}{l} x[n] \leftrightarrow X(z) \\ h[n] \leftrightarrow H(z)\end{array}\right\}$ $(x*h)[n] \leftrightarrow H(z)X(z)$, su RdC <u>contiene</u> $RdC_x \cap RdC_h$

Como $H(z)$ tiene 2 polos, hay 3 posibles RdC: a) $|z|<1/4$, b) $1/4<|z|<3/4$, c) $|z|>3/4$

La region correcta se determina aplicando la propiedad:



- Si RdC_H es $|z|<1/4$, se intersecta con RdC_x, pero RdC_y <u>NO</u> la contiene $\Rightarrow$ <u>NO</u> es compatible
- Si RdC_H es $1/4<|z|<3/4$, se intersecta con RdC_x, y RdC_y lo contiene $\Rightarrow$ ES compatible
- Si RdC_H es $|z|>3/4$, se intersecta con RdC_x, y la RdC_y lo contiene $\Rightarrow$ Tb es compatible

$\Rightarrow$ Hay 2 posibles RdC para $H(z)$ que son compatibles con $X(z)$ e $Y(z)$ dados

- $1/4<|z|<3/4$
- $|z|>3/4$

Cálculo de $h[n]$: $H(z) = \frac{(1-1/8 z^{-1})(1-7/8 z^{-1})}{(1-1/4 z^{-1})(1-3/4 z^{-1})} = \frac{1 - z^{-1} + \frac{7}{64} z^{-2}}{1 - z^{-1} + \frac{3}{16} z^{-2}}$ mismo orden ⇒ división

$$\begin{array}{l|l} \frac{7}{64} z^{-2} - z^{-1} + 1 & \frac{3}{16} z^{-2} - z^{-1} + 1 \\ \frac{7}{64} z^{-2} - \frac{7}{12} z^{-1} + \frac{7}{12} & \frac{7}{12} \\ \hline -\frac{5}{12} z^{-1} + \frac{5}{12} & \end{array}$$

$$H(z) = \frac{7}{12} + \frac{-\frac{5}{12} z^{-1} + \frac{5}{12}}{(1-1/4 z^{-1})(1-3/4 z^{-1})} = \frac{7}{12} + \frac{A}{(1-1/4 z^{-1})} + \frac{B}{(1-3/4 z^{-1})}$$

$$A = \left.\frac{(1-1/8 z^{-1})(1-7/8 z^{-1})}{(1-1/4 z^{-1})(1-3/4 z^{-1})} (1-1/4 z^{-1})\right|_{z=1/4} = \frac{5}{8}$$

$$B = \left.\frac{(1-1/8 z^{-1})(1-7/8 z^{-1})}{(1-1/4 z^{-1})(1-\cancel{3/4} z^{-1})} (1-\cancel{\tfrac{3}{4}} z^{-1})\right|_{z=3/4} = -\frac{5}{24}$$

$$H(z) = \frac{7}{12} + \frac{5}{8} \frac{1}{1-1/4 z^{-1}} - \frac{5}{24} \frac{1}{1-3/4 z^{-1}}$$

- $|z| > 3/4$ ⇒ las sucesiones asociadas a los polos son derechas

$$h_1[n] = \frac{7}{12} \delta[n] + \frac{5}{8} \left(\frac{1}{4}\right)^n u[n] - \frac{5}{24} \left(\frac{3}{4}\right)^n u[n]$$

- $\frac{1}{4} < |z| < \frac{3}{4}$ ⇒ la sucesión asociada al polo en $1/4$ es derecha, la asociada a $3/4$ es izquierda

$$h_2[n] = \frac{7}{12} \delta[n] + \frac{5}{8} \left(\frac{1}{4}\right)^n u[n] + \frac{5}{24} \left(\frac{3}{4}\right)^n u[-n-1]$$

$x[n] = \cos(\omega_0 n)$, $\quad 0 \leq n \leq N-1$, $\quad N = 8$, $\quad \omega_0 = \frac{3}{8} \cdot 2\pi = \frac{3}{4}\pi$

a) $X(e^{j\omega}) = ?$

1ª forma: $\quad x[n] = x_1[n] \cdot w[n]$, $\quad$ donde $\quad x_1[n] = \cos(\omega_0 n)$, $\quad -\infty < n < \infty$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad w[n] = 1$, $\quad 0 \leq n \leq N-1$

$\downarrow$

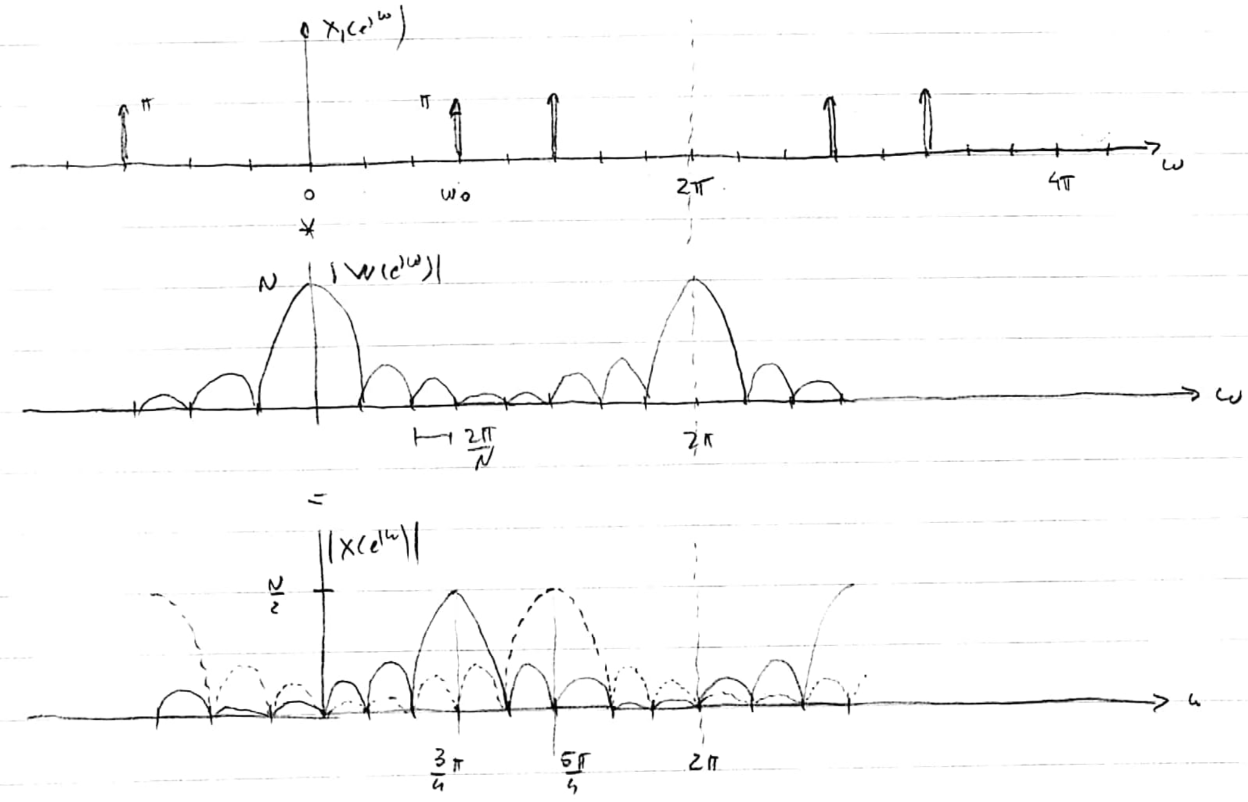
$$X(e^{j\omega}) = \text{"} X_1(e^{j\omega}) * W(e^{j\omega}) \text{"} = \frac{1}{2\pi} \int_{2\pi} W(e^{j\theta}) X_1(e^{j(\omega-\theta)}) d\theta$$

$$X_1(e^{j\omega}) = \sum_k \pi\, \delta(\omega - \omega_0 - 2\pi k) + \pi\, \delta(\omega + \omega_0 - 2\pi k)$$

$$W(e^{j\omega}) = \sum_{n=0}^{N-1} 1\, e^{-j\omega n} = \frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}} = \frac{e^{-j\omega \frac{N}{2}} \left(e^{j\omega \frac{N}{2}} - e^{-j\omega \frac{N}{2}}\right)}{e^{-j\omega/2} \left(e^{j\omega/2} - e^{-j\omega/2}\right)} = e^{-j\frac{N-1}{2}\omega} \frac{\operatorname{sen}(N\omega/2)}{\operatorname{sen}(\omega/2)}$$

$$\Rightarrow X(e^{j\omega}) = \frac{1}{2\pi} \int_{2\pi} W(e^{j\theta}) X(e^{j(\omega-\theta)}) d\theta = \frac{1}{2} W(e^{j(\omega-\omega_0)}) + \frac{1}{2} W(e^{j(\omega+\omega_0)})$$

$$\boxed{X(e^{j\omega}) = \frac{1}{2} e^{-j\frac{N-1}{2}(\omega-\omega_0)} \frac{\operatorname{sen}[N(\omega-\omega_0)/2]}{\operatorname{sen}((\omega-\omega_0)/2)} + \frac{1}{2} e^{-j\frac{N-1}{2}(\omega+\omega_0)} \frac{\operatorname{sen}[N(\omega+\omega_0)/2]}{\operatorname{sen}[(\omega+\omega_0)/2]}}$$



(No dibujo la suma)

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2nd Form:

$$x[n] = \cos\omega_0 n, \qquad 0 \le n \le N-1$$

$$= \frac{1}{2}e^{j\omega_0 n} + \frac{1}{2}e^{-j\omega_0 n}$$

$$X(e^{j\omega}) = \sum_{n=0}^{N-1}\left(\frac{1}{2}e^{j\omega_0 n}\right)e^{-j\omega n} + \sum_{n=0}^{N-1}\left(\frac{1}{2}e^{-j\omega_0 n}\right)e^{-j\omega n}$$

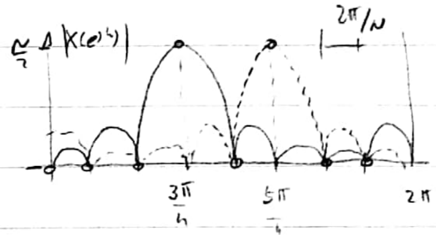
$$= \sum_{n=0}^{N-1}\frac{1}{2}e^{-j(\omega-\omega_0)n} + \sum_{n=0}^{N-1}\frac{1}{2}e^{-j(\omega+\omega_0)N}$$

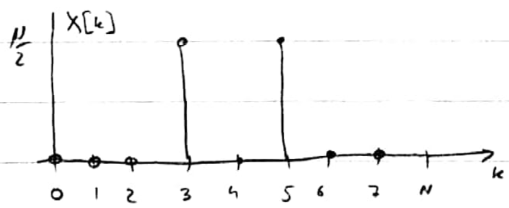
$$= \frac{1}{2}\,\frac{e^{-j(\omega-\omega_0)N}-1}{e^{-j(\omega-\omega_0)}-1} + \frac{1}{2}\,\frac{e^{-j(\omega+\omega_0)N}-1}{e^{-j(\omega+\omega_0)}-1}$$

$$= \frac{1}{2}\,\frac{e^{-j(\omega-\omega_0)\frac{N}{2}}\left(e^{-j(\omega-\omega_0)\frac{N}{2}} - e^{j(\omega-\omega_0)\frac{N}{2}}\right)}{e^{-j(\omega-\omega_0)/2}\left(e^{-j\frac{\omega-\omega_0}{2}} - e^{j\frac{\omega-\omega_0}{2}}\right)} + \frac{1}{2}\,\frac{e^{-j(\omega+\omega_0)\frac{N}{2}}\left(e^{-j(\omega+\omega_0)\frac{N}{2}} - e^{j(\omega+\omega_0)\frac{N}{2}}\right)}{e^{-j\frac{\omega+\omega_0}{2}}\left(e^{-j\frac{\omega+\omega_0}{2}} - e^{j\frac{\omega+\omega_0}{2}}\right)}$$

$$X(e^{j\omega}) = \frac{1}{2}e^{-j\frac{N-1}{2}(\omega-\omega_0)}\,\frac{\sin[N(\omega-\omega_0)/2]}{\sin[(\omega-\omega_0)/2]} + \frac{1}{2}e^{-j\frac{N-1}{2}(\omega+\omega_0)}\,\frac{\sin[N(\omega+\omega_0)/2]}{\sin[(\omega+\omega_0)/2]}$$

b) $X[k] = X(e^{j\omega})\big|_{\omega=\frac{2\pi}{N}k}, \qquad 0 \le k \le N-1$





$$X[k] = \frac{1}{2}e^{-j\frac{7}{2}\left(\frac{2\pi}{8}k-\frac{6\pi}{8}\right)}\,\frac{\sin\left(8\left(\frac{2\pi}{8}k-\frac{6\pi}{8}\right)/2\right)}{\sin\left[\left(\frac{2\pi}{8}k-\frac{6\pi}{8}\right)/2\right]} + \frac{1}{2}e^{-j\frac{7}{2}\left(\frac{2\pi}{8}k+\frac{6\pi}{8}\right)}\,\frac{\sin\left[8\left(\frac{2\pi}{8}k+\frac{6\pi}{8}\right)/2\right]}{\sin\left[\left(\frac{2\pi}{8}k+\frac{6\pi}{8}\right)/2\right]}$$

$$X[k] = \frac{1}{2}e^{-j\frac{7}{8}\pi(k-3)}\,\frac{\sin[(k-3)\pi]}{\sin[(k-3)\pi/8]} + \frac{1}{2}e^{-j\frac{7}{8}\pi(k+3)}\,\frac{\sin[(k+3)\pi]}{\sin[(k+3)\pi/8]}$$

$$\frac{\operatorname{sen}[(k-3)\pi]}{\operatorname{sen}[(k-3)\pi/8]} = 0 \ \forall k \text{ salvo } k=3 \text{ (se anula numerador y denominador) donde vale 8 ("l'Hopital")}$$

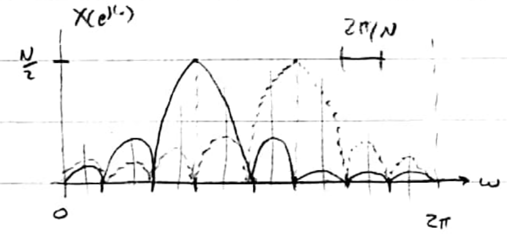
$$\frac{\operatorname{sen}[(k+3)\pi]}{\operatorname{sen}[(k+3)\pi/8]} = 0 \ \forall k \text{ salvo para } k=5 \text{ (se anula numerador y denominador) donde vale 8 ("l'Hopital")}$$

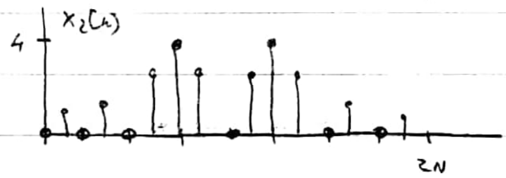
$\rightarrow$ en el rango $0 \leq k \leq N-1 \Rightarrow$

$$\Rightarrow \quad X[k] = \begin{cases} 4, & \text{si } k=3 \text{ o } k=5 \\ 0, & \text{en caso contrario} \end{cases}$$

c) $X_2[k]$ = la TDF de $2N$ puntos de $x[n]$

$$X_2[k] = X(e^{j\omega})\Big|_{\omega = \frac{2\pi}{2N}k}, \quad k = 0 \ldots 2N-1$$





$$X_2[k] = \frac{1}{2} e^{-j\frac{N-1}{2}(\omega-\omega_0)} \frac{\operatorname{sen}[N(\omega-\omega_0)/2]}{\operatorname{sen}[(\omega-\omega_0)/2]} + \frac{1}{2} e^{-j\frac{N-1}{2}(\omega+\omega_0)} \frac{\operatorname{sen}[N(\omega+\omega_0)/2]}{\operatorname{sen}[(\omega+\omega_0)/2]} \Bigg|_{\omega = \frac{2\pi}{2N}k = \frac{\pi}{8}k}$$

$$= \frac{1}{2} e^{-j\frac{7}{2}\left(\frac{\pi}{8}k - \frac{3\pi}{4}\right)} \frac{\operatorname{sen}\left[8\left(\frac{\pi}{8}k - \frac{3\pi}{4}\right)/2\right]}{\operatorname{sen}\left[\left(\frac{\pi}{8}k - \frac{3\pi}{4}\right)/2\right]} + \frac{1}{2} e^{-j\frac{7}{2}\left(\frac{\pi}{8}k + \frac{3\pi}{4}\right)} \frac{\operatorname{sen}\left[8\left(\frac{\pi}{8}k + \frac{3\pi}{4}\right)/2\right]}{\operatorname{sen}\left[\left(\frac{\pi}{8}k + \frac{3\pi}{4}\right)/2\right]}$$

$$X_2[k] = \frac{1}{2} e^{-j\frac{7}{16}\pi(k-6)} \frac{\operatorname{sen}\left[\frac{\pi}{2}(k-6)\right]}{\operatorname{sen}\left[\frac{\pi}{16}(k-6)\right]} + \frac{1}{2} e^{-j\frac{7}{16}\pi(k+6)} \frac{\operatorname{sen}\left[\frac{\pi}{2}(k+6)\right]}{\operatorname{sen}\left[\frac{\pi}{16}(k+6)\right]}, \quad k = 0 \ldots 15$$

- la TDF se anula en los "k" para $\left[(k \pm 6)\frac{\pi}{2} \text{ es múltiplo de } \pi\right]$ salvo en $k=6$ y en $k=N-6$
- la TDF no se anula en k impar

6

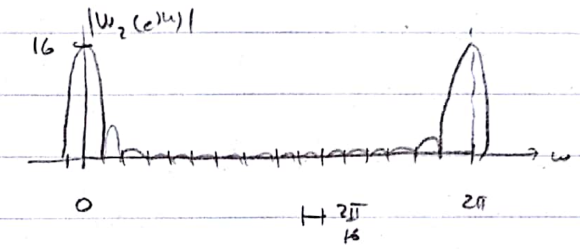
d) $X_3[k]$, la TDF de $x_3[n] = \{x_1[n], x_1[n]\}$

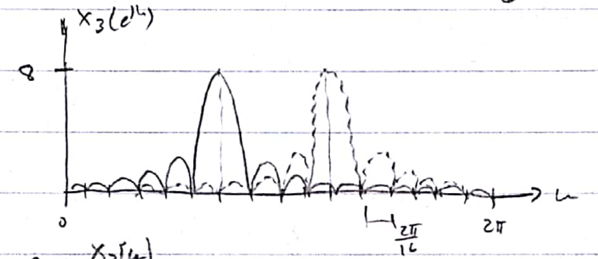
Se puede pensar en forma similar al inciso (c), pero consideramos $w[n] = \begin{cases} 1, & 0 \le n \le 2N-1 \\ 0, & cc \end{cases}$

$$W_2(e^{j\omega}) = \sum_{n=0}^{2N-1} 1\, e^{-j\omega n} = \frac{e^{-j(2N)\omega}-1}{e^{-j\omega}-1} = \frac{e^{-jN\omega}\left(e^{-jN\omega}-e^{jN\omega}\right)}{e^{-j\omega/2}\left(e^{-j\omega/2}-e^{j\omega/2}\right)} = e^{-j\frac{2N-1}{2}\omega}\,\frac{\sin(2N\omega/2)}{\sin\omega/2}$$

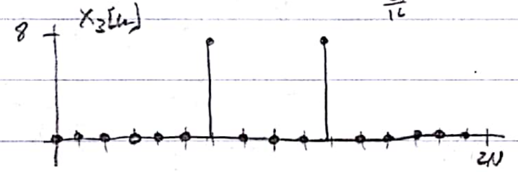
Para $N=8$,

$$W_2(e^{j\omega}) = e^{-j\frac{15}{2}\omega}\,\frac{\sin(8\omega)}{\sin(\omega/2)}$$





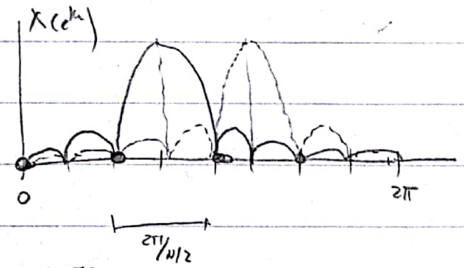
$$X_3[k] = X_3(e^{j\omega})\Big|_{\omega=\frac{2\pi}{2N}k} = \frac{\pi}{8}k$$

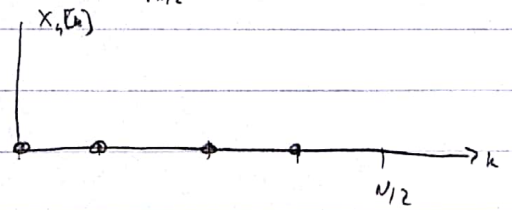


$$X_3[k] = \begin{cases} 8, & k=6,\ k=10 \\ 0, & cc \end{cases}$$

e) 1ra opción: $X_4[k] = X(e^{j\omega})\Big|_{\omega=\frac{2\pi}{N/2}k}$

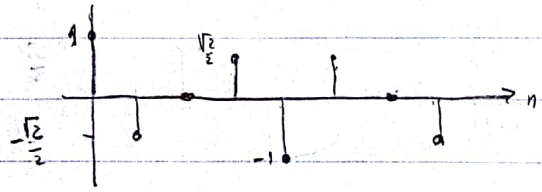
$\Rightarrow X_4[k] = 0 \quad \forall k \quad (k=0,1,2,3)$

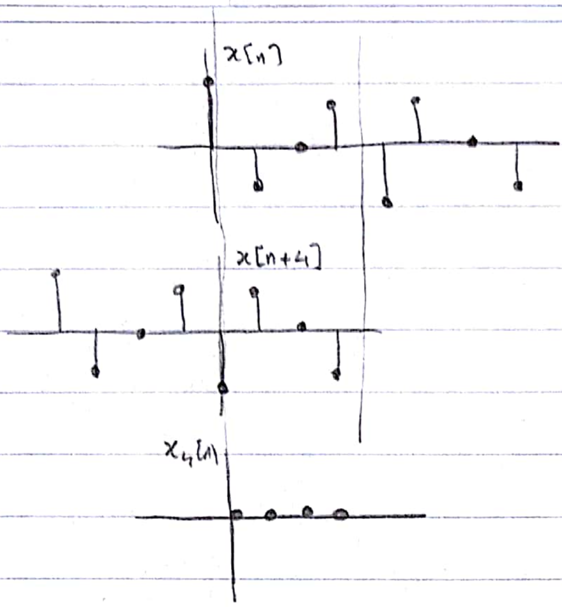




f) $x_4[n] = \sum_r x_1[n - rN/2], \quad 0 \le n < \frac{N}{2}$ $\qquad x[n] = \cos\frac{3\pi}{4}n$

$= x[n] + x[n+\frac{N}{2}], \quad 0 \le n < \frac{N}{2}$



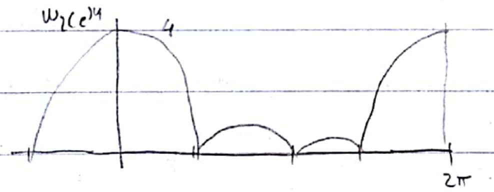


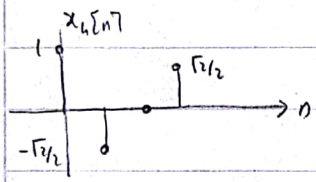
—x—

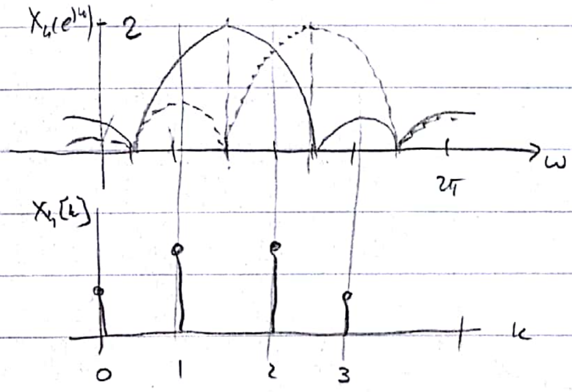
c) Este inciso podría interpretarse de esta forma:

$x_4[n] = \cos(\omega_0 n) \quad 0 \le n \le \frac{N}{2}-1$ En este caso es similar al inciso (a)

con $w_2[n] = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & cc \end{cases} \Rightarrow W_2(e^{j\omega}) = e^{-j\frac{3}{2}\omega} \dfrac{sen(4\omega/2)}{sen(\omega/2)}$







$$X_4(e^{j\omega}) = \frac{1}{2}W_2(e^{j(\omega-\omega_0)}) + \frac{1}{2}W_2(e^{j(\omega+\omega_0)}) = \frac{1}{2}e^{-j\frac{3}{2}(\omega-\omega_0)}\frac{sen[4(\omega-\omega_0)/2]}{sen[(\omega-\omega_0)/2]} + \frac{1}{2}e^{-j\frac{3}{2}(\omega+\omega_0)}\frac{sen[4(\omega+\omega_0)/2]}{sen[(\omega+\omega_0)/2]}$$

$$X_4[k] = X_4(e^{j\omega})\Big|_{\omega = \frac{2\pi}{4}k = \frac{\pi}{2}k}$$

$$X_4[k] = \frac{1}{2}e^{-j\frac{3}{2}(\frac{\pi}{2}k - \frac{3\pi}{4})}\frac{sen[4(\frac{\pi}{2}k-\frac{3\pi}{4})/2]}{sen[(\frac{\pi}{2}k-\frac{3\pi}{4})/2]} + \frac{1}{2}e^{-j\frac{3}{2}(\frac{\pi}{2}k+\frac{3\pi}{4})}\frac{sen[4(\frac{\pi}{2}k+\frac{3\pi}{4})/2]}{sen[(\frac{\pi}{2}k+\frac{3\pi}{4})/2]}$$

$$X_4[k] = \frac{1}{2}e^{-j\frac{3\pi}{4}(k-3/2)}\frac{sen[(k-3/2)\pi]}{sen[(k-3/2)\pi/4]} + \frac{1}{2}e^{-j\frac{3\pi}{4}(k+\frac{3}{2})}\frac{sen[(k+3/2)\pi]}{sen[(k+3/2)\pi/4]}$$ $\Rightarrow$ no se anula para $k=0,1,2,3$.